Massive MIMO for Maximum Spectral Efficiency

Mérouane Debbah

www.huawei.com

Mathematical and Algorithmic Sciences Lab

HUAWEI TECHNOLOGIES CO., LTD.

Before 2010…Random Matrices and MIMO

Some Results on the Asymptotic Downlink Capacity of MIMO Multi-user Networks

Raul L. de Lacerda Neto and Mérouane Debbah
Mobile Communications Group, Institut Eurecom
2229 Route des Cretes, B.P. 193, 06904 Sophia Antipolis, France
E-mail: {Raul.De-Lacerda,Debbah}@eurecom.fr

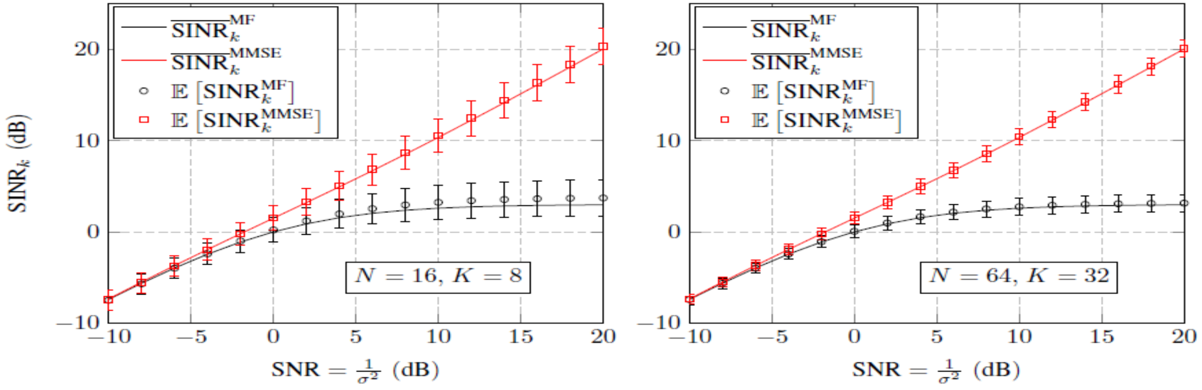
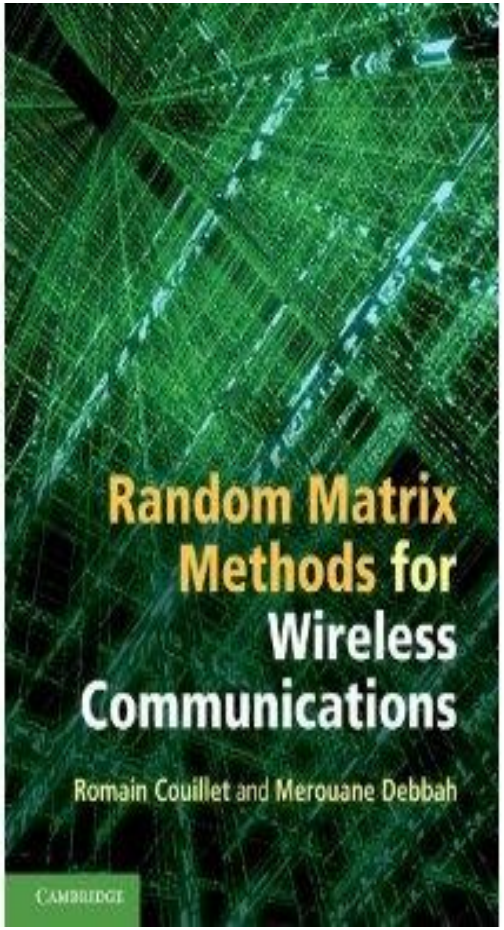
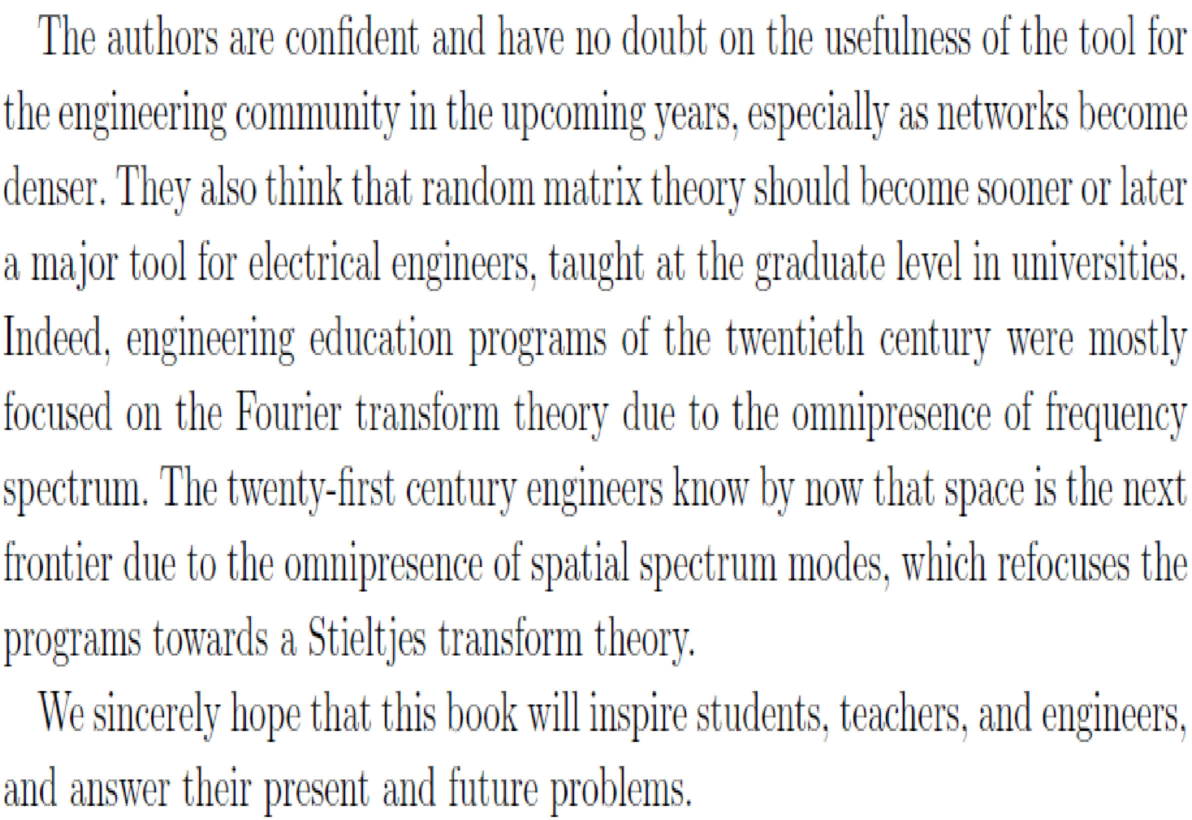


Figure: Errorbars correspond to one standard deviation in each direction.

The asymptotic results are quite accurate for reasonable values of N, K.

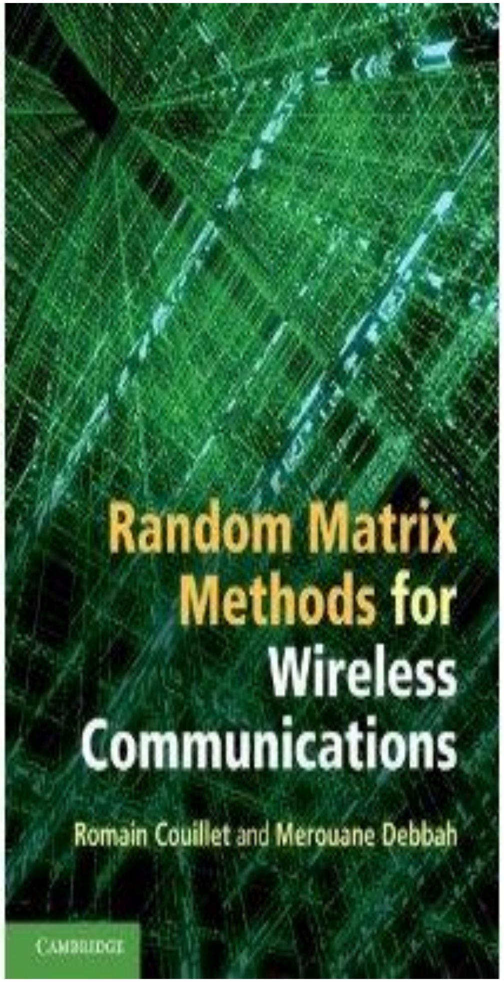


Random Matrices and MIMO



The authors are confident and have no doubt on the usefulness of the tool for the engineering community in the upcoming years, especially as networks become denser. They also think that random matrix theory should become sooner or later a major tool for electrical engineers, taught at the graduate level in universities. Indeed, engineering education programs of the twentieth century were mostly focused on the Fourier transform theory due to the omnipresence of frequency spectrum. The twenty-first century engineers know by now that space is the next frontier due to the omnipresence of spatial spectrum modes, which refocuses the programs towards a Stieltjes transform theory.

We sincerely hope that this book will inspire students, teachers, and engineers, and answer their present and future problems.



Beyond LTE: The 400-Antenna Base Station

Thomas L. Marzetta
Bell Laboratories
Alcatel-Lucent

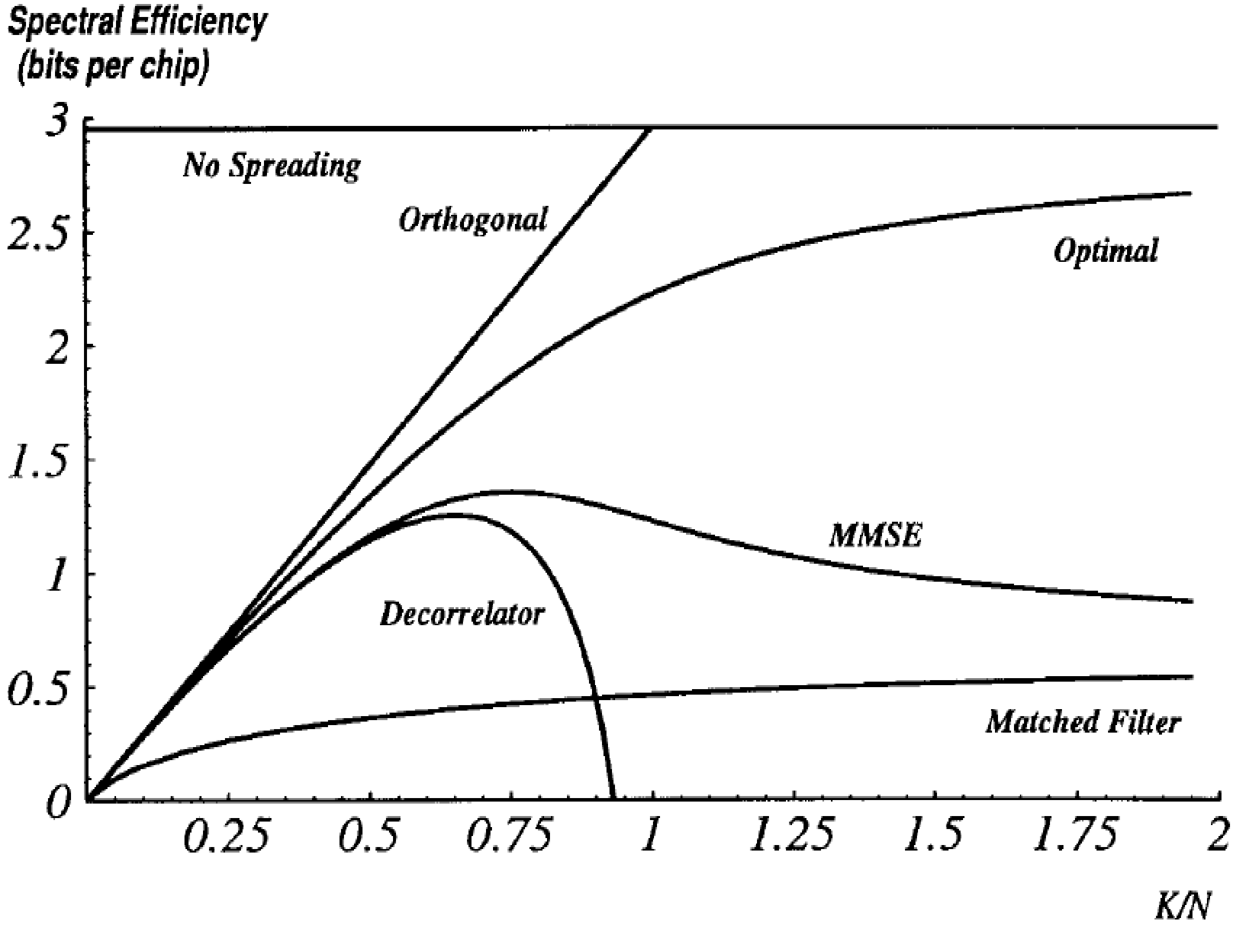
28 May, 2010

Green Touch Initiative

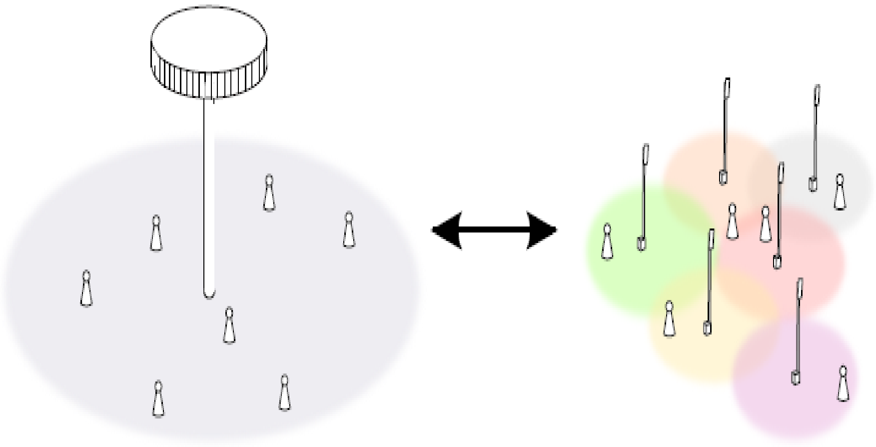
The Challenge

Demonstrate technologies in 5 years that will lead to a **1,000-fold reduction** in energy consumption

J. Hoydis, S. ten Brink, M. Debbah, "*Massive MIMO in the UL/DL of Cellular Networks: How Many Antennas Do We Need?*," IEEE Journal on Selected Areas in Communications, 2013. **IEEE Leonard G. Abraham Prize**



“David vs Goliath“ or "Small Cells vs Massive MIMO“



How to densify: *“More antennas or more BSs?”*

Questions:

- Should we install more base stations or simply more antennas per base?
- How can massively many antennas be efficiently used?

Example

- Density of UTs: $\lambda_{UT} = 16$
- Constant transmit power density: $P \times \lambda_{BS} = 10$
- Number of BS-antennas: $N = \lambda_{UT}/\lambda_{BS}$
- Path loss exponent: $\alpha = 4$
- UT simultaneously served on each band: $K = \lambda_{UT}/(\lambda_{BS} \times L)$

$\Rightarrow$ Only two parameters: λ_{BS} and L

Table: Average spectral efficiency C/W in (bits/s/Hz)

sub-bands L	$\lambda_{BS} = 1$	$\lambda_{BS} = 2$	$\lambda_{BS} = 4$	$\lambda_{BS} = 8$	$\lambda_{BS} = 16$
1	0.6209	0.8188	1.1964	1.5215	2.1456
2	1.1723	1.2414	1.3404	1.5068	x
4	0.8882	0.8973	1.1964	x	x
8	0.5689	0.5952		x	x
16	0.3532	x	x	x	x

Fully distributing the antennas gives highest throughput gains!

5G

- 1-10Gbps connections to end points in the field (i.e. not theoretical maximum)
- 1 millisecond end-to-end round trip delay (latency)
- 1000x bandwidth per unit area
- 10-100x number of connected devices
- (Perception of) 99.999% availability
- (Perception of) 100% coverage
- 90% reduction in network energy usage
- Up to ten year battery life for low power, machine-type devices

Massive MIMO as one of the operating of 5G

E. Bjornson, L. Sanguinetti, J. Hoydis and M. Debbah, "Designing Multi-User MIMO for Energy Efficiency: When is Massive MIMO the Answer? », IEEE Wireless Communications and Networking Conference (WCNC) 2014, Istanbul, Turkey, **BEST PAPER AWARD.**

The Three Phases of Massive MIMO

Every great scientific truth goes through three phases.

- 1) First, people deny it.
- 2) Second, they say it conflicts with the physics (engineering) principles
- 3) Third, they say they’ve known it all along.

Typical Statements about Massive MIMO

- “*Massive MIMO improves spectral efficiency with orders of magnitude*”
  - This sounds promising but is vague!
  - Which gains can we expect in reality?

- “*Massive MIMO has an order of magnitude more antennas than users*”
  - This assumption reduces interference
  - But does it maximize any system performance metric?

- “*The pilot sequences are reused for channel estimation in every cell*”
  - This is an analytically tractable assumption
  - Are there no benefits of having more pilot sequences than that?

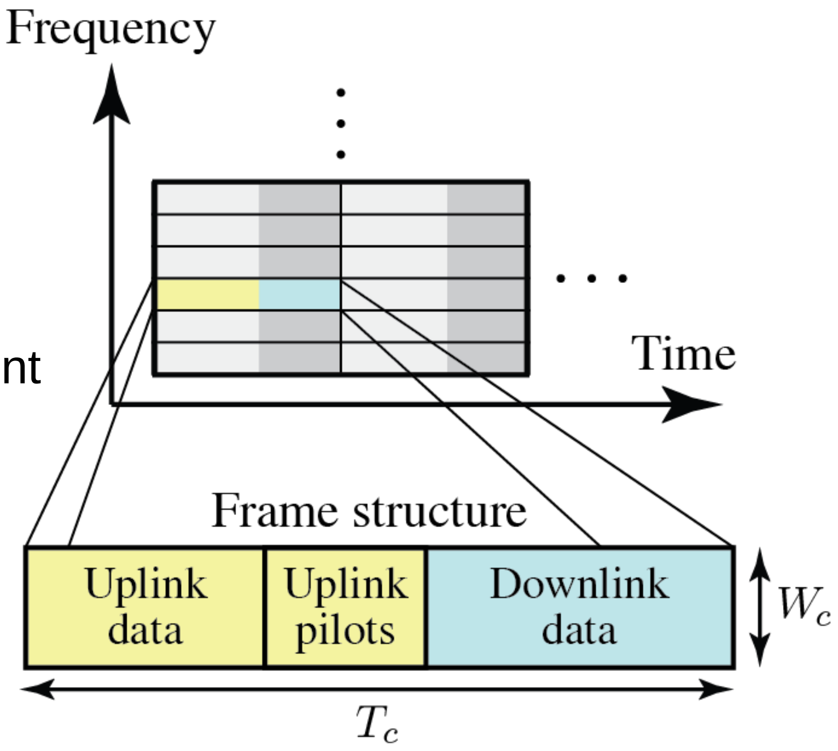
Partial Answers in This Talk!

Goal: Optimize spectral efficiency for a given number of antennas

Variables: Number of users and pilot sequences

Massive MIMO Transmission Protocol

- Coherence Blocks
  - Fixed channel responses
  - Coherence time: T_c s
  - Coherence bandwidth: W_c Hz
  - Depends on mobility and environment
  - Block length: $\tau_c = T_c W_c$ symbols
  - Typically: $\tau_c \in [100,10000]$



- Time-Division Duplex (TDD)
  - Downlink and uplink on all frequencies
  - τ_p symbols/block for uplink pilots – for channel estimation
  - $\tau_c - \tau_p$ symbols/block for uplink and/or downlink payload data

Linear or Non-linear Processing?

- Capacity-Achieving Non-linear Processing
  - Downlink: Dirty paper coding
  - Uplink: Successive interference cancellation

Do we need it in Massive MIMO?

Linear Processing

Bad when $M \approx K$

Good when $M/K > 2$

Relative low complexity

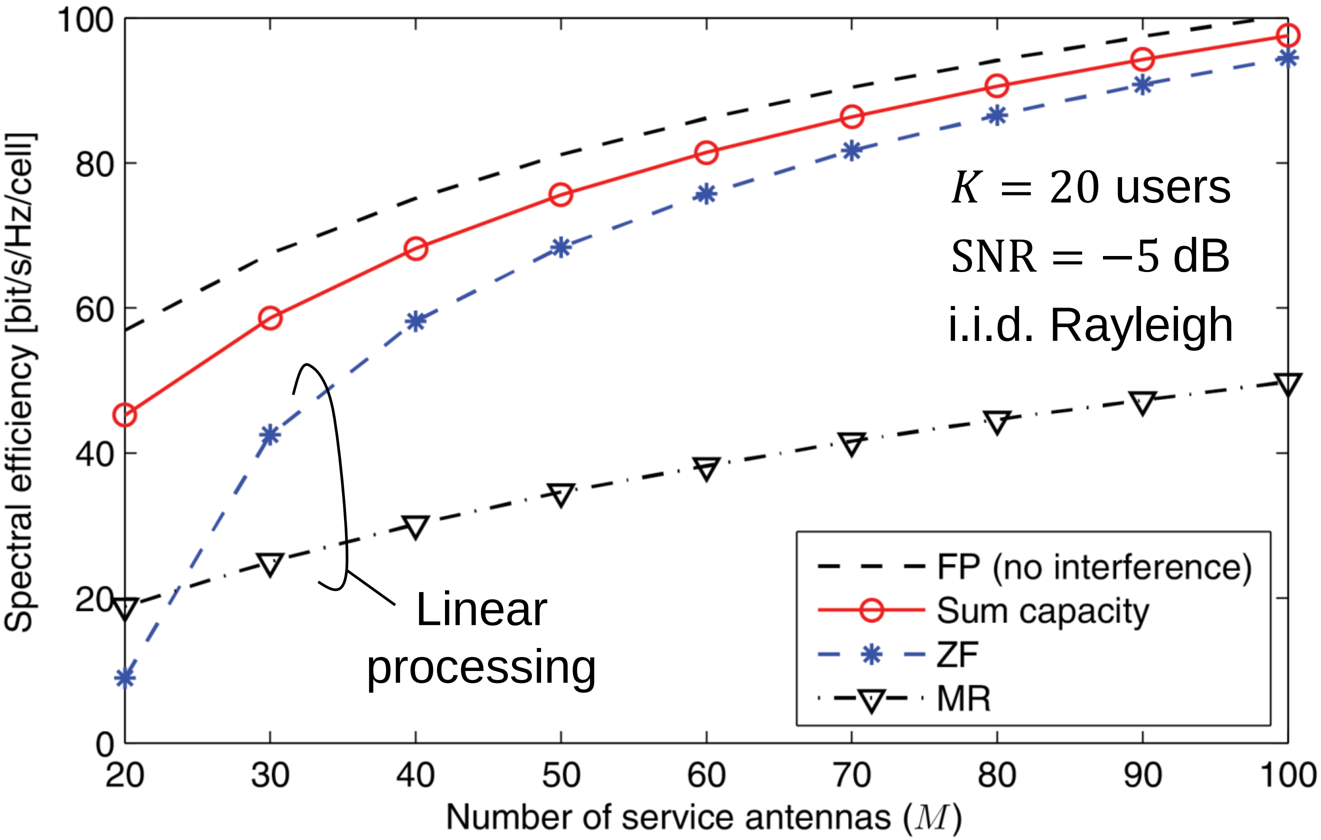
Massive MIMO

Uses linear processing:

Maximum ratio (MR)

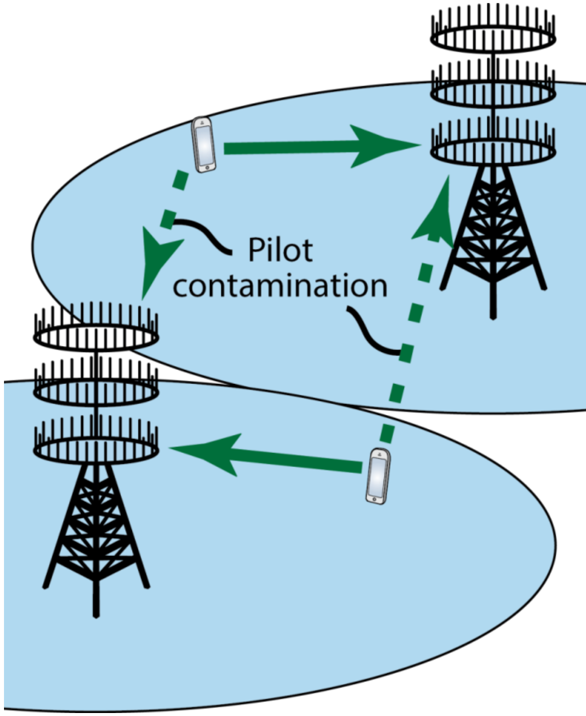
Zero-forcing (ZF)

MMSE



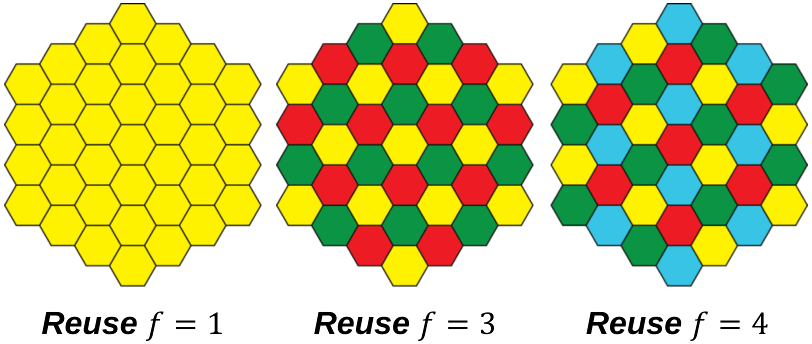
Channel Acquisition in Massive MIMO

- Limited Number of Pilots: $\tau_p \leq \tau_c$
  - Must use same pilot sequence in several cells
  - Base stations cannot tell some users apart: *Essence of pilot contamination*

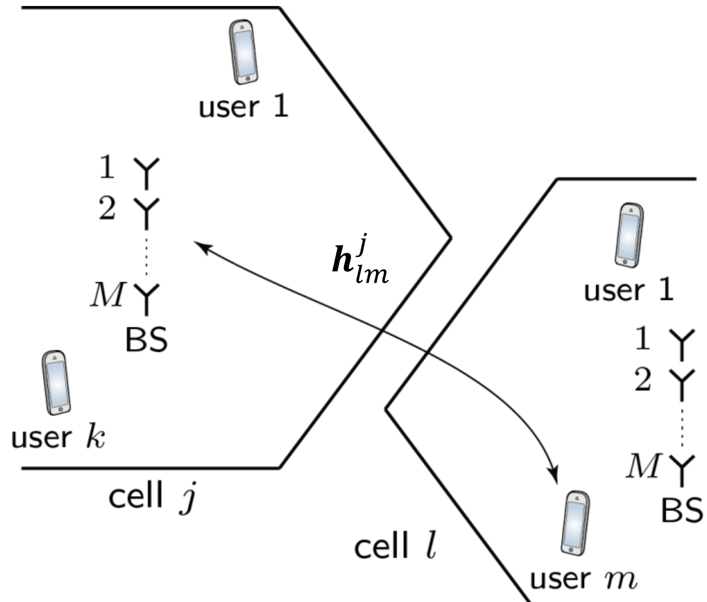


- Coordinated Pilot Allocation
  - Allocate pilots to users to reduce contamination
  - Scalability → No signaling between BSs

- Solution: Non-universal pilot reuse
  - Pilot reuse factor $f \geq 1$
  - Users per cell: $K = \frac{\tau_p}{f}$
  - $\mathcal{P}_j(f)$: Cells with same pilots as BS j
  - Higher f → Fewer users per cell, but fewer interferers in $\mathcal{P}_j$



Basic Spectral Efficiency Expressions (1/3)



- System Model
  - Channel from BS j to user m in cell l

$$\boldsymbol{h}_{lm}^{j} \sim \mathcal{CN}\left(\mathbf{0}, \beta_{lm}^{j} \mathbf{I}_M\right)$$

  - Uplink transmit power: ρ_{jk}^{u}
  - Downlink transmit power: ρ_{jk}^{d}

- Channel Estimation Quality at BS l

$$\gamma_{lm}^{j} = \frac{\rho_{lm}^{u} \beta_{lm}^{j} \tau_p}{\sum_{l' \in \mathcal{P}_l(f)} \rho_{l'm}^{u} \beta_{l'm}^{j} \tau_p + \sigma^2}$$

(Note: $0 \leq \gamma_{lm}^{j} \leq 1$)

  - MMSE estimate distribution:

$$\widehat{\boldsymbol{h}}_{lm}^{j} \sim \mathcal{CN}\left(\mathbf{0}, \beta_{lm}^{j} \gamma_{lm}^{j} \mathbf{I}_M\right)$$

Estimate error distribution:

$$\boldsymbol{e}_{lm}^{j} \sim \mathcal{CN}\left(\mathbf{0}, \beta_{lm}^{j} (1 - \gamma_{lm}^{j}) \mathbf{I}_M\right)$$

Basic Spectral Efficiency Expressions (2/3)

***Recall:** Spectral efficiency of AWGN channel*

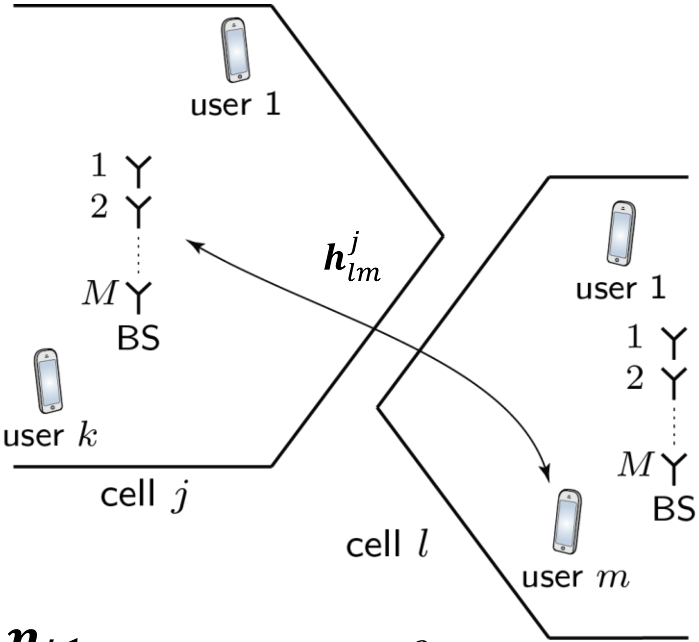
$$y = g \cdot s + n$$

Constant gain

Signal: $CN(0, \rho)$

Noise: $CN(0, \sigma^2)$

$$R = \log_2\left(1 + \frac{\rho |g|^2}{\sigma^2}\right)$$



Stochastic uplink channel in cell j:

$$\boldsymbol{y}_j = \sum_{l,m} \boldsymbol{h}_{lm}^{j} s_{lm} + \boldsymbol{n}_j$$

Noise: $CN(\boldsymbol{0}, \sigma^2 \boldsymbol{I})$

Signal: $s_{lm} \sim CN(0, \rho_{lm}^{u})$

- Linear detector $\boldsymbol{v}_{jk}$ for user k in cell j:

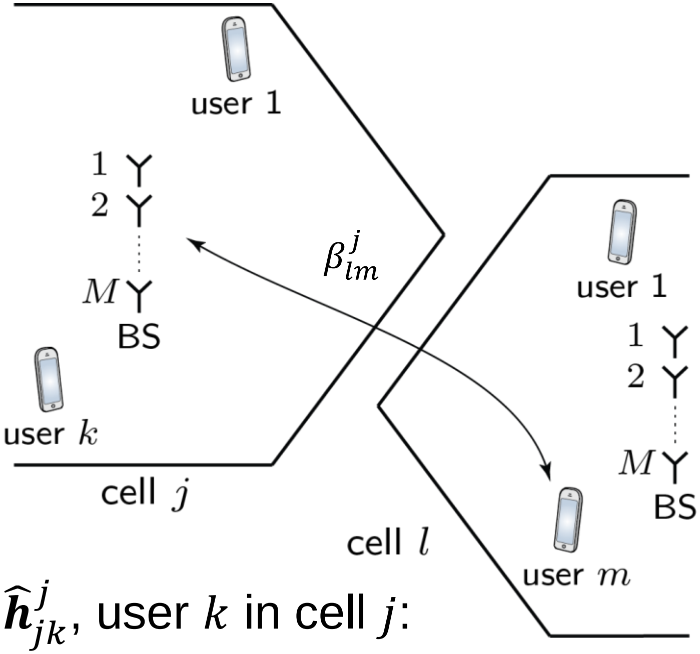
$$\boldsymbol{v}_{jk}^{H} \boldsymbol{y}_j = \underbrace{\mathbb{E}\left\{\boldsymbol{v}_{jk}^{H} \widehat{\boldsymbol{h}}_{jk}^{j}\right\}}_{g_{jk}\text{: Known constant gain}} s_{jk} + \underbrace{\left(\boldsymbol{v}_{jk}^{H} \boldsymbol{h}_{jk}^{j} - \mathbb{E}\left\{\boldsymbol{v}_{jk}^{H} \widehat{\boldsymbol{h}}_{jk}^{j}\right\}\right) s_{jk}}_{a_{jk}\text{: Signal along unknown direction}} + \underbrace{\sum_{(l,m) \neq (j,k)} \boldsymbol{v}_{jk}^{H} \boldsymbol{h}_{lm}^{j} s_{lm} + \boldsymbol{v}_{jk}^{H} \boldsymbol{n}_j}_{b_{jk}\text{: Multi-user interference and noise}}$$

Lower bound, mutual info: $\log_2\left(1 + \frac{\rho_{jk}^{u} |g_{jk}|^2}{\mathbb{E}\left\{|a_{jk}|^2\right\} + \mathbb{E}\left\{|b_{jk}|^2\right\}}\right)$

Basic Spectral Efficiency Expressions (3/3)

- Lower Bound on Spectral Efficiency
  - Averaged over small-scale fading
  - Depends on variance β^j_{lm}
  - Depends on estimation quality:

$$\gamma^j_{lm} = \frac{\rho^u_{lm}\beta^j_{lm}\tau_p}{\sum_{l'\in\mathcal{P}_l(f)}\rho^u_{l'm}\beta^j_{l'm}\tau_p+\sigma^2}$$



  - Uplink spectral efficiency with MR, $\boldsymbol{v}_{jk} = \hat{\boldsymbol{h}}^j_{jk}$, user k in cell j:

Array gain

$$R_{jk} = \underbrace{\left(1-\frac{\tau_p}{\tau_c}\right)}_{\textit{Pilot overhead}} \log_2\left(1+\frac{\overbrace{M\,\rho^u_{jk}\,\beta^j_{jk}\,\gamma^j_{jk}}^{\textit{Desired signal}}}{\underbrace{\sum_{l,m}\rho^u_{lm}\,\beta^j_{lm}}_{\textit{Conventional interference}} + M\underbrace{\sum_{l\in\mathcal{P}_j(f)\setminus\{j\}}\rho^u_{lk}\beta^j_{lk}\,\gamma^j_{lk}}_{\textit{Pilot-contaminated interference}}+\sigma^2}\right)$$

Similar expressions for downlink and with ZF processing

Optimization of Spectral Efficiency

- How Large Spectral Efficiency can be Achieved?
  - Problem Formulation:

$$\underset{K,\,\tau_p}{\text{maximize}} \quad \text{total spectral efficiency} \qquad [\text{bit/s/Hz/cell}]$$

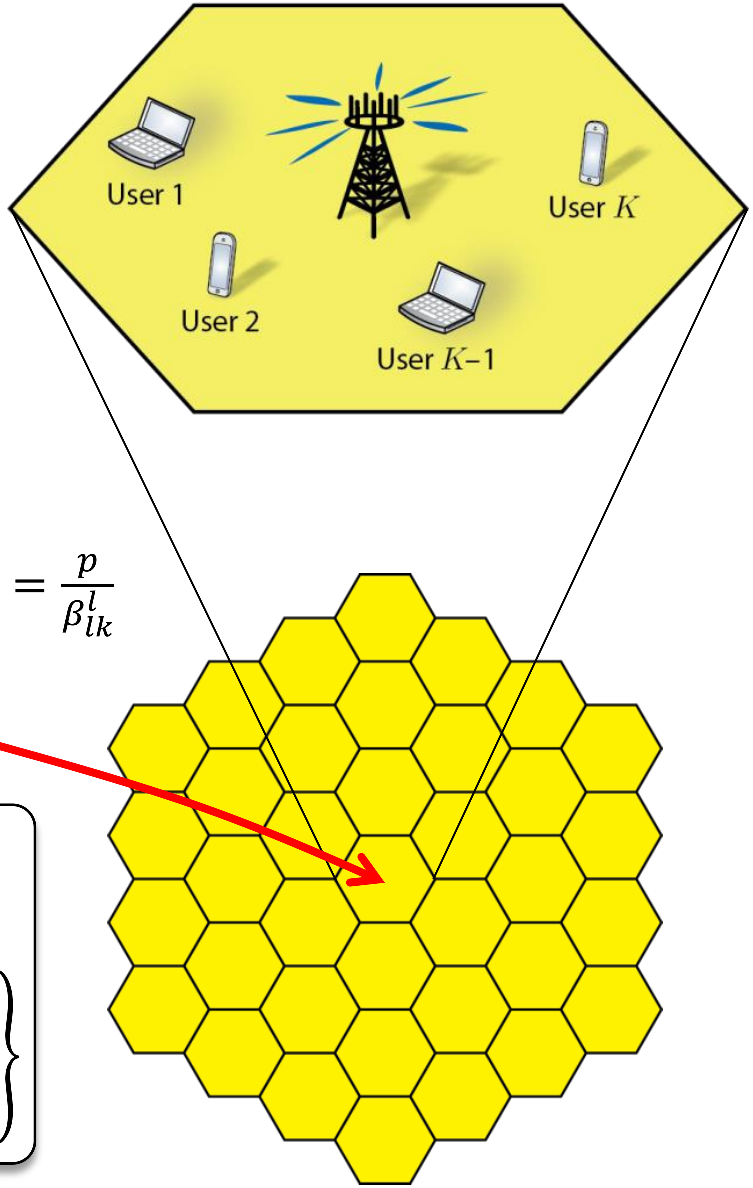
for a given M *and* τ_c.

- Issue: Hard to use previous expressions
  - Interference depends on all users' positions! (i.e., on all β_{lm}^{j})
  - We want quantitative results – averaged over user locations
  - We want to avoid non-informative Monte-Carlo simulations

- Solution: Make every user "typical"
  - Same uplink SNR: Power control inversely proportional to pathloss
  - Inter-cell interference: Average over interfering user locations in other cells

Symmetric Multi-Cell Network

- Classic Hexagonal Network
  - Infinite grid of hexagonal cells
  - M antennas at each BS
  - K active users in each cell
  - Same user distribution in each cell
  - Uncorrelated Rayleigh fading
  - Statistical uplink channel inversion: $\rho_{lk}^{u} = \frac{p}{\beta_{lk}^{l}}$



Every cell is “typical”

Propagation Parameters

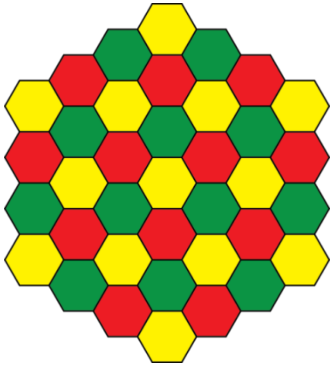
(Average interference from cell l to BS j)

Compute $\mu_{jl}^{(1)} = \mathbb{E}\left\{\frac{\beta_{lk}^{j}}{\beta_{lk}^{l}}\right\}$ *and* $\mu_{jl}^{(2)} = \mathbb{E}\left\{\left(\frac{\beta_{lk}^{j}}{\beta_{lk}^{l}}\right)^{2}\right\}$

Coordinated Precoding and Detection

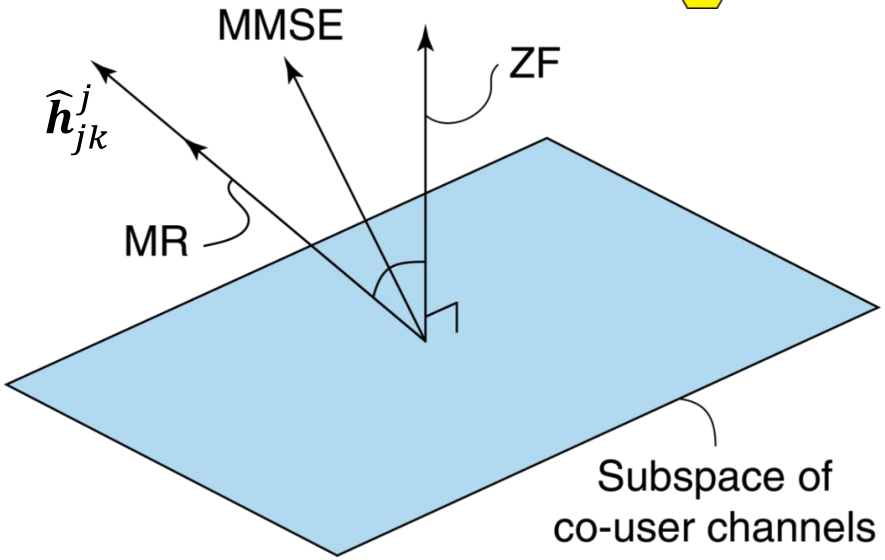
- Coordinated Multi-Point (CoMP)
  - Avoid causing strong inter-cell interference
  - Scalability → No signaling between BSs
- Solution: Observe and react ($f \geq 1$)
  - Listen to pilot signals used only in other cells
  - Utilize to suppress inter-cell interference
  - Schemes: Multi-cell ZF and multi-cell MMSE

Reuse $f = 3$



MMSE precoding/detection:

$$\boldsymbol{v}_{lk} = \Big(\underbrace{\sum_{l,m} \rho^{u}_{lm}\hat{\boldsymbol{h}}^{j}_{lm}\big(\hat{\boldsymbol{h}}^{j}_{lm}\big)^{H}}_{\textit{All estimated channels}} + \underbrace{\boldsymbol{E}_j}_{\textit{Estimation error covariance matrix}} + \sigma^2\boldsymbol{I}\Big)^{-1}\hat{\boldsymbol{h}}^{j}_{jk}$$



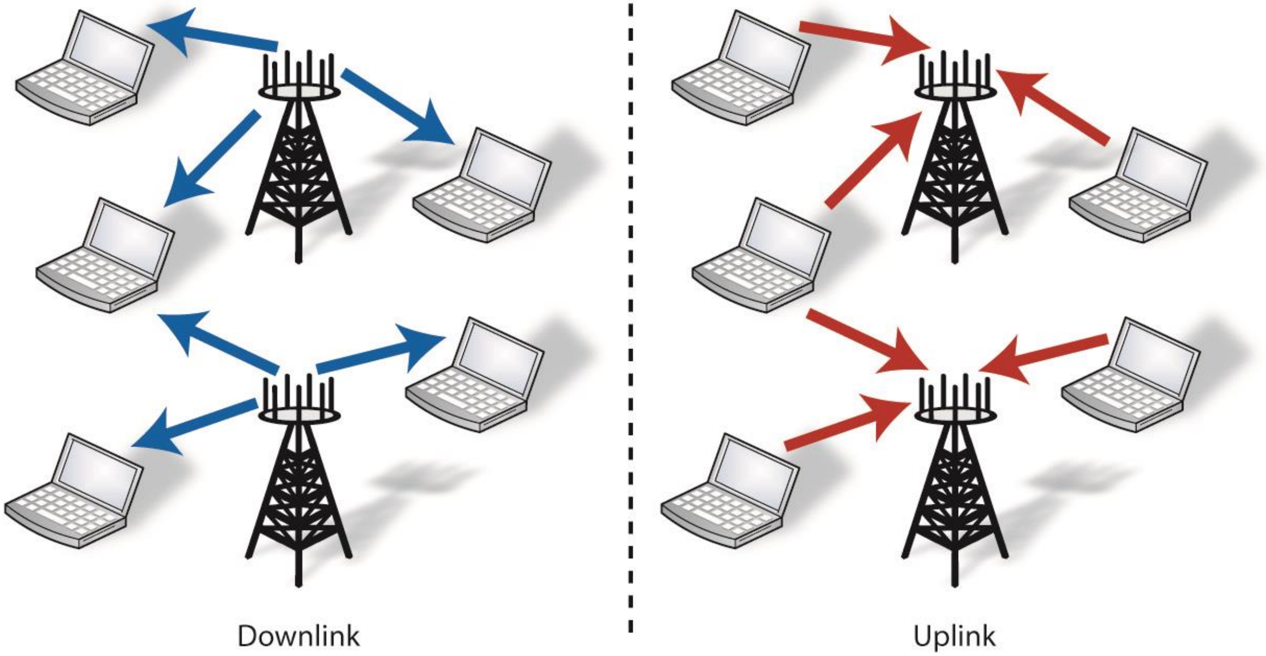
Uplink-Downlink Duality

Duality Theorem

Any set of uplink SEs is also achievable in the downlink using same sum power

Same precoding/detection vectors, but different power allocation

Note: Equivalence between two lower bounds – uplink bound is looser!



Average Spectral Efficiency per Cell (1/2)

- Lower Bound on Average Ergodic Capacity in Cell j:

$$\mathrm{SE}_j = \mathbb{E}\left\{ \sum_{k=1}^{K} \mathrm{SE}_{jk} \right\} = K\left(1 - \frac{\tau_p}{\tau_c}\right) \underbrace{\mathbb{E}\{\log_2(1 + \mathrm{SINR}_{jk})\}}_{\substack{\text{Every user is “typical”:} \\ \text{Same for all users}}}$$

$$\geq K\left(1 - \frac{\tau_p}{\tau_c}\right) \log_2\left(1 + \frac{1}{\mathbb{E}\{\mathrm{SINR}_{jk}^{-1}\}}\right)$$

Jensen’s inequality

Remaining expectation can be computed explicitly for:

MR, ZF, and M-multi-cell ZF (M-ZF)

Average Spectral Efficiency per Cell (2/2)

- Lower Bound on Average Ergodic Capacity in Cell j:

$$\mathrm{SE}_j = K \underbrace{\left(1 - \frac{\tau_p}{\tau_c}\right)}_{\text{Loss from pilots}} \log_2\left(1 + \underbrace{\frac{1}{I_j}}_{\text{“SINR”}}\right)$$

- Interference term depends on processing:

$$I_j^{\mathrm{MR}} = \underbrace{\sum_{l\in\mathcal{P}_j(f)\setminus\{j\}} \left(\boxed{\mu_{jl}^{(2)}} + \frac{\mu_{jl}^{(2)} - (\mu_{jl}^{(1)})^2}{M}\right)}_{\text{Pilot contamination}} + \underbrace{\left(\frac{\sum_{l\in\mathcal{L}} \mu_{jl}^{(1)} K + \frac{\sigma^2}{p}}{M}\right)}_{\text{Interference from all cells}} \underbrace{\left(\sum_{l\in\mathcal{P}_j(f)} \mu_{jl}^{(1)} + \frac{\sigma^2}{p\tau_p}\right)}_{\text{1/(Estimation quality)}}$$

$$I_j^{\mathrm{ZF}} = \overbrace{\sum_{l\in\mathcal{P}_j(f)\setminus\{j\}} \left(\boxed{\mu_{jl}^{(2)}} + \frac{\mu_{jl}^{(2)} - (\mu_{jl}^{(1)})^2}{M-K}\right)} + \overbrace{\left(\frac{\sum_{l\in\mathcal{L}} \mu_{jl}^{(1)} K + \frac{\sigma^2}{p}}{M-K}\right)} \overbrace{\left(\sum_{l\in\mathcal{P}_j(f)} \mu_{jl}^{(1)} + \frac{\sigma^2}{p\tau_p}\right)} - \overbrace{\sum_{l\in\mathcal{P}_j(f)} \frac{\left(\mu_{jl}^{(1)}\right)^2 K}{M-K}}^{\text{Interference suppression}}$$

Only terms that remain as $M \to \infty$*: Finite limit on SE*

Asymptotic Limit on Spectral Efficiency

- Lower Bound on Average Ergodic Capacity as $M \to \infty$:

$$\mathrm{SE}_j \to K\left(1 - \frac{fK}{\tau_c}\right)\log_2\left(1 + \frac{1}{\sum_{l \in \mathcal{P}_j(f)\setminus\{j\}} \mu_{jl}^{(2)}}\right)$$

How Many Users to Serve?

Pre-log factor $K\left(1 - \frac{fK}{\tau_c}\right)$ *is maximized by* $K^* = \frac{\tau_c}{2f}$ *users*

Maximal SE: $\frac{\tau_c}{4f}\log_2\left(1 + \frac{1}{\sum_{l \in \mathcal{P}_j(f)\setminus\{j\}} \mu_{jl}^{(2)}}\right)$

Try different f *and* $\mathcal{P}_j(f)$ *to maximize the limit*

How Long Pilot Sequences?

$\tau_p = fK^* = \frac{\tau_c}{2}$: *Spend half coherence interval on pilots!*

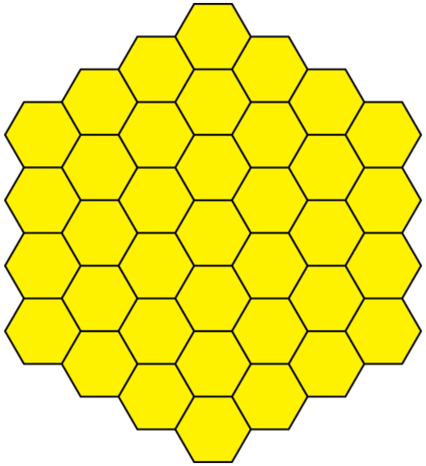
Numerical Results

- Problem Formulation:

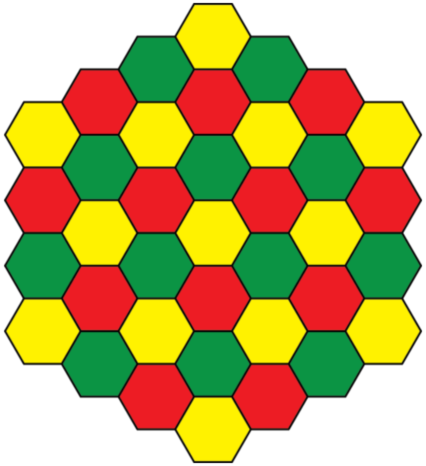
$$\underset{K,\tau_p}{\text{maximize}} \quad \text{total spectral efficiency} \qquad [\text{bit/s/Hz/cell}]$$

for a given M *and* τ_c.

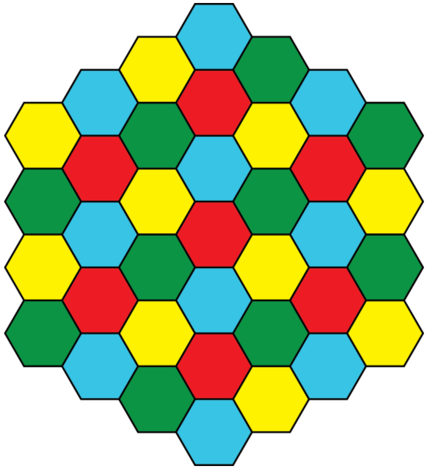
  - Use average spectral efficiency expressions
  - Compute average interference $\mu_{jl}^{(1)}$ and $\mu_{jl}^{(2)}$ (a few minutes)
  - Compute for different K and f and pick maximum ($<$ 1 minute)



Reuse $\boldsymbol{f = 1}$



Reuse $\boldsymbol{f = 3}$



Reuse $\boldsymbol{f = 4}$

Assumptions

Pathloss exponent: 3.7

Coherence: $\tau_c = 400$

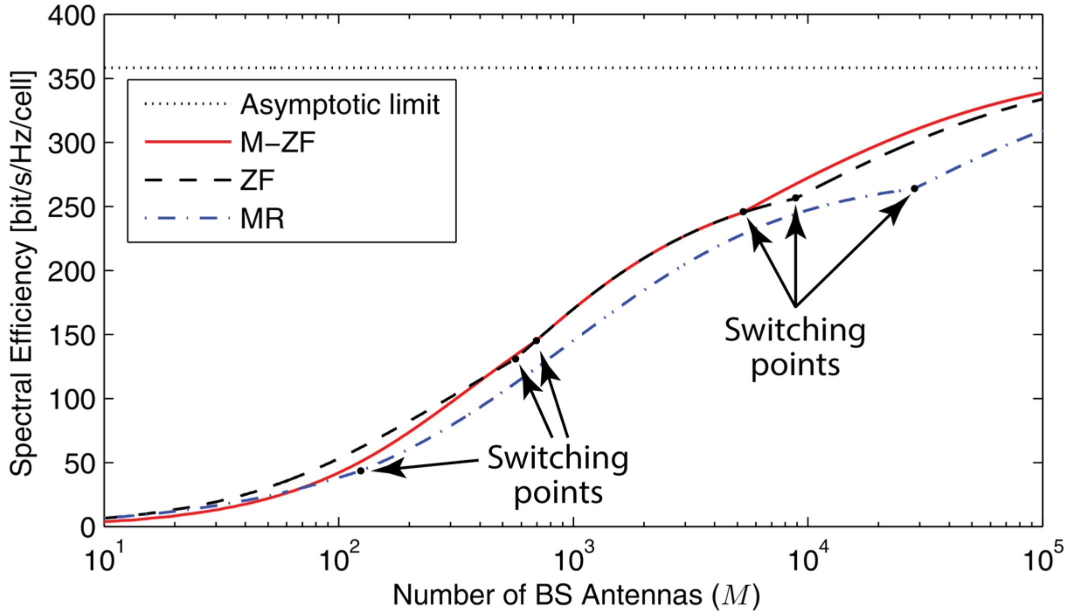
Rayleigh fading

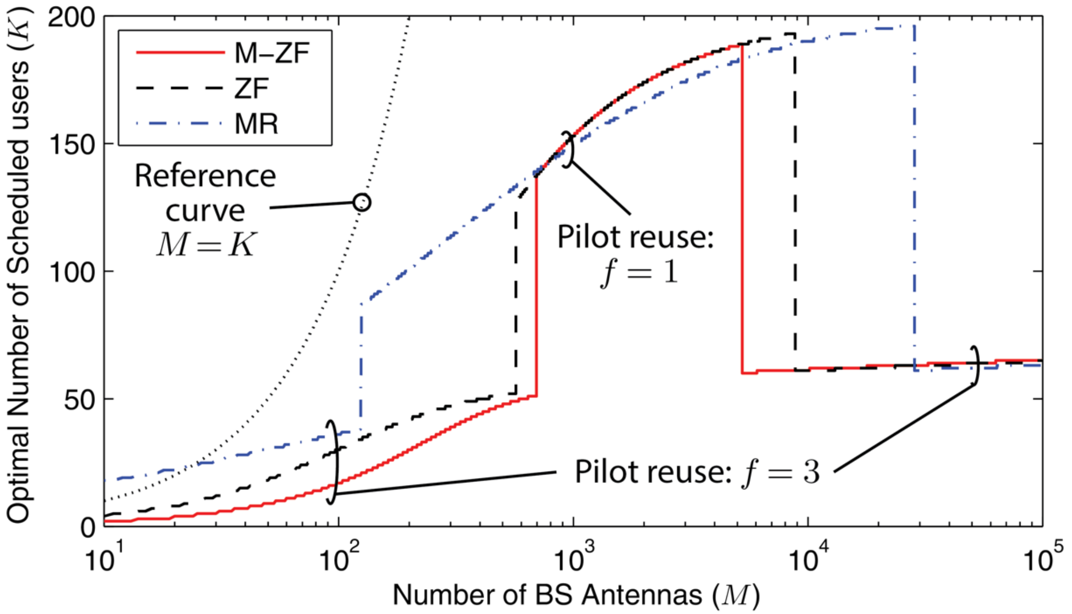
SNR 5 dB

Asymptotic Behavior: Mean-Case Interference

Observations

- *Uniform user distributions*
- *Asymptotic limits not reached*
- *Reuse factor $f = 3$ is desired*
- *K is different for each scheme*
- *Small difference between optimized schemes*
- *Coordinated beamforming: Better at very large M*

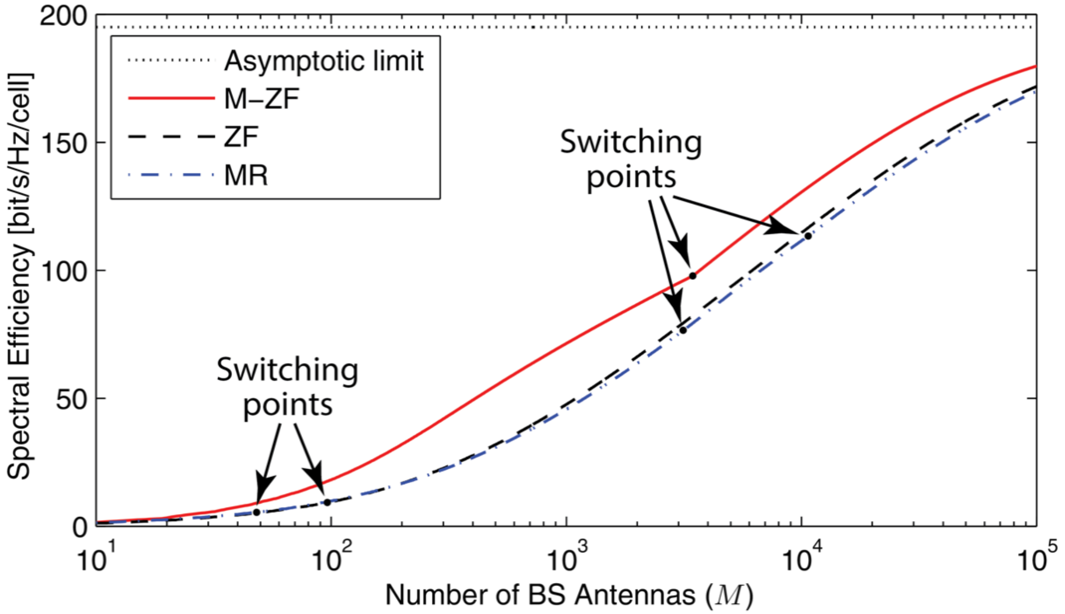


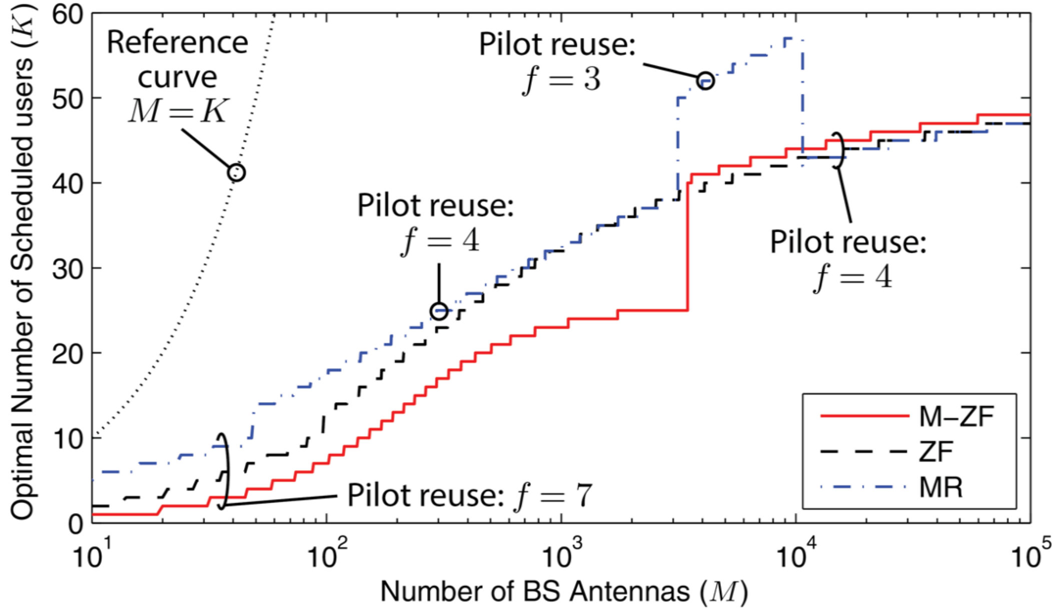


Asymptotic Behavior: Worst-Case Interference

Observations

- *Interferers at worst positions*
- *Asymptotic limits not reached*
- *Reuse factor $f = 4$ is desired*
- *K is different for each scheme*
- *Coordinated beamforming: Brings large gains for all M*





Flexible Number of Users

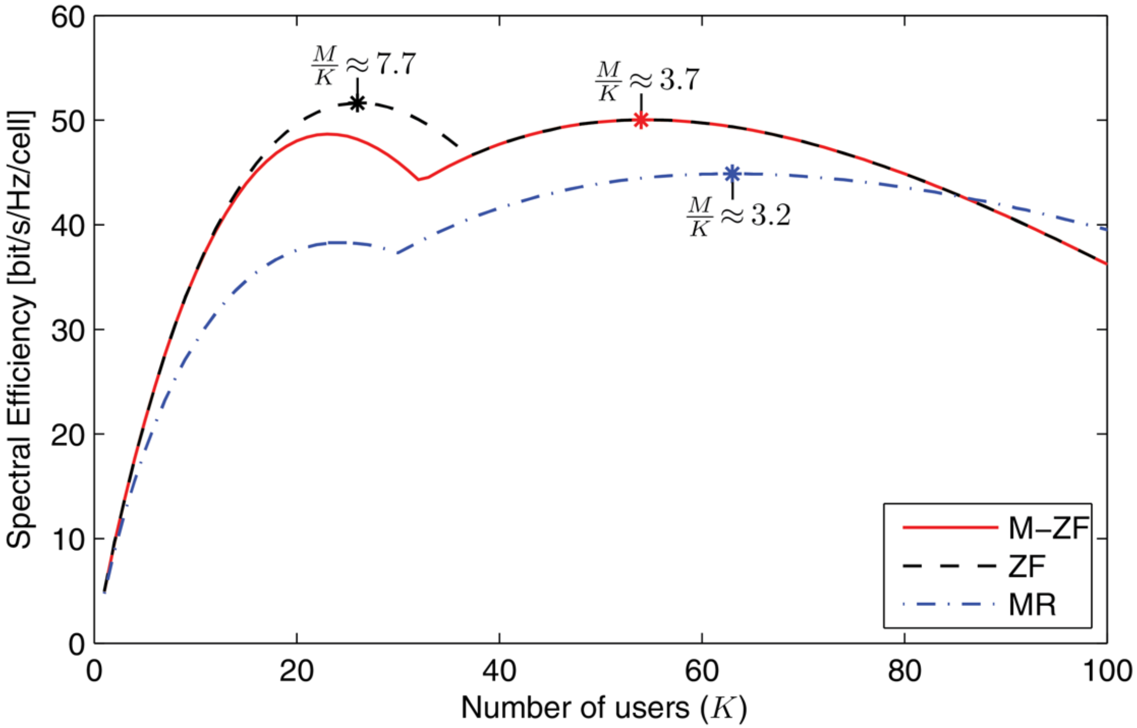
- SE w.r.t. number of users ($M = 200$ antennas)
  - Mean-case interference
  - Optimized reuse factors
  - Equal SNR (5 dB)

Observations

Stable SE for $K > 10$:
Trivial scheduling:
Admit everyone

M-ZF, ZF, and MR provide similar per-cell performance

$M/K < 10$ is just fine!



Spectral Efficiency per User

- User Performance for Optimized System
  - Mean-case interference
  - Optimized reuse factors
  - Equal SNR (5 dB)

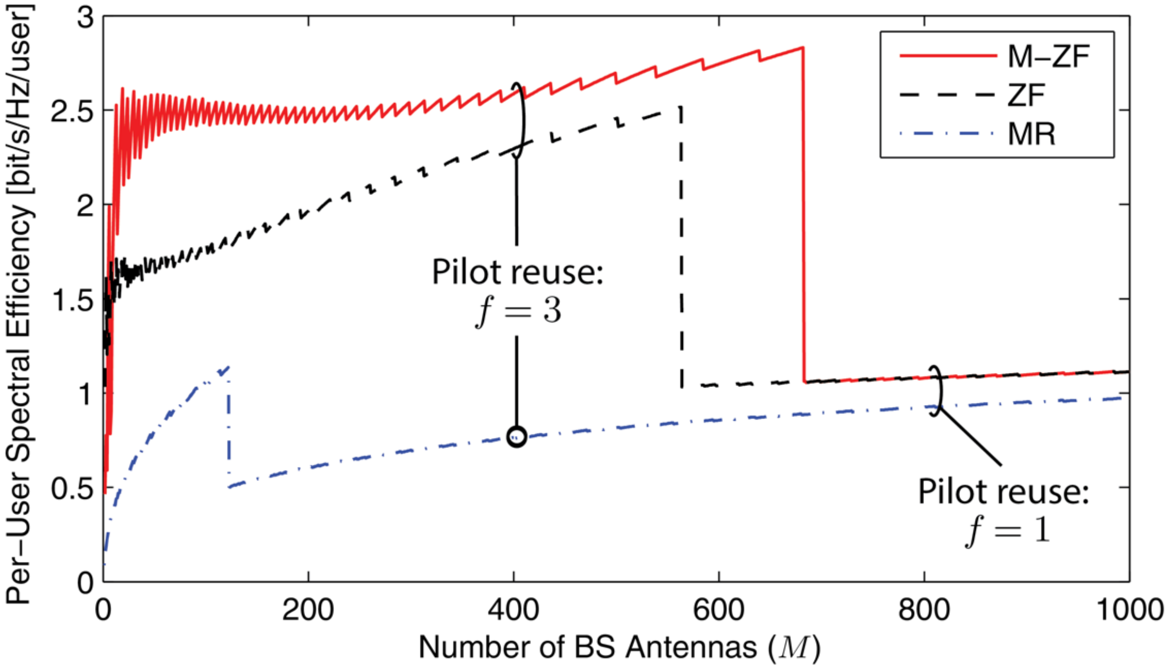
Observations

User performance is modest:

BPSK, Q-PSK, or 16-QAM

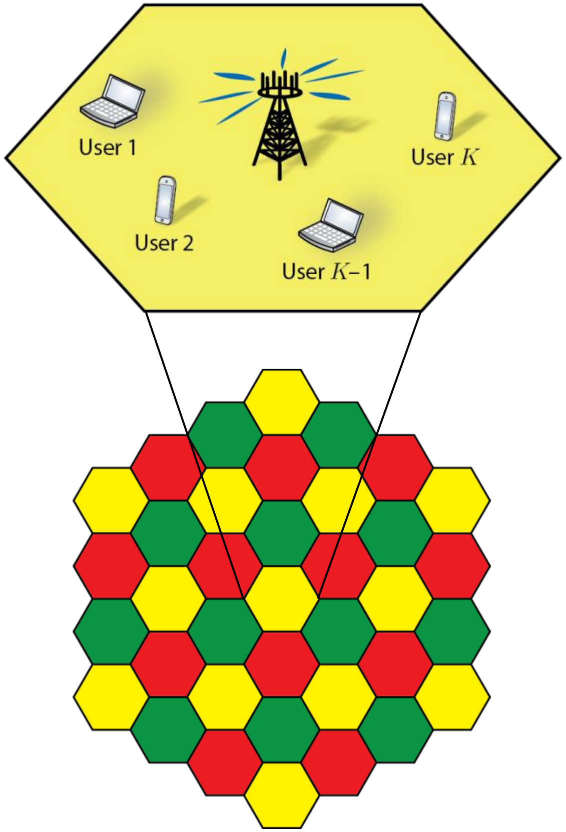
Schemes for different purposes:

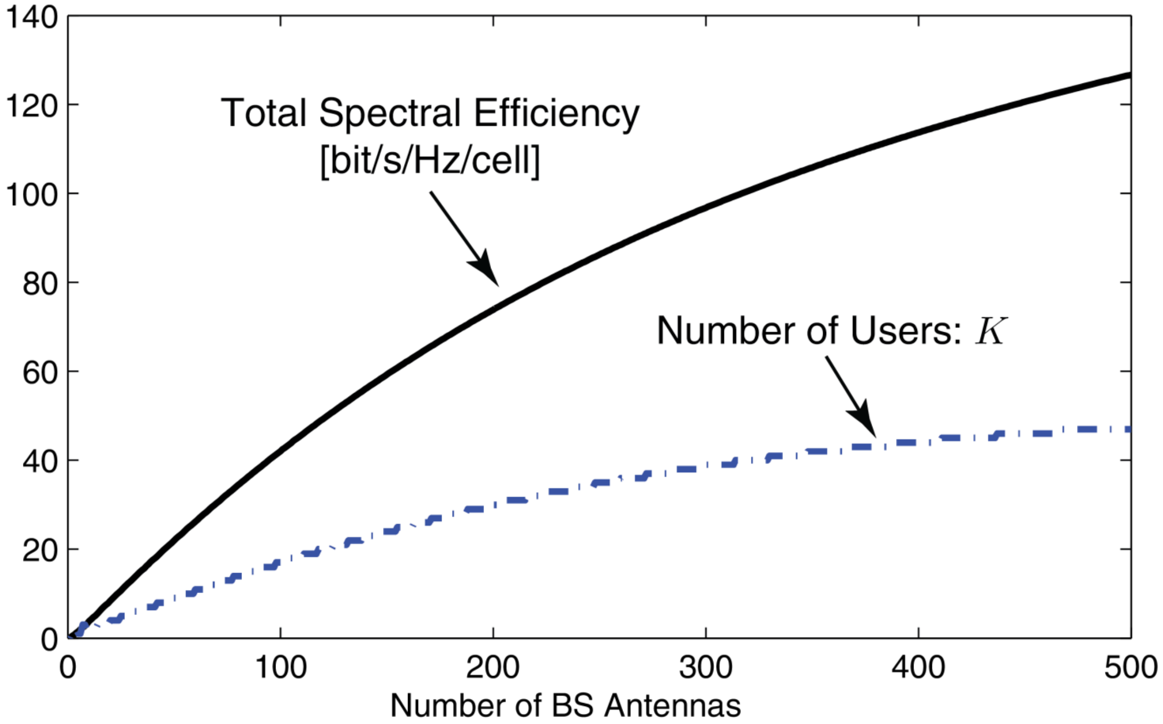
M-ZF > ZF > MR



Also applicable in the downlink!

Anticipated Uplink Spectral Efficiency





Assumptions

ZF processing

Pilot reuse: $f = 3$

Observations

- *Baseline: 2.25 bit/s/Hz/cell (IMT-Advanced)*
- *Massive MIMO,* $M = 100$*: x20 gain* $(M/K \approx 6)$
- *Massive MIMO,* $M = 400$*: x50 gain* $(M/K \approx 9)$
- *Per scheduled user:* ≈ 2.5 *bit/s/Hz*

Summary

- Massive MIMO delivers High Spectral Efficiency
  - $> 20\text{x}$ gains over IMT-Advanced are within reach
  - Very high spectral efficiency per cell, not per user
  - Non-universal pilot reuse ($f = 3$) is often preferred
  - MR, ZF, M-ZF prefer different values on K and f
  - “An order of magnitude more antennas than users” is not needed

- Asymptotic limits
  - Coherence interval (τ_c symbols) limits multiplexing capability
  - Allocate up to $\tau_c/2$ symbols for pilots
  - We can handle very many users/cell – how many will there be?

Key References (1/2)

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